磁場對承受定值表面熱與質量通量的雙分層多孔介質內垂直圓錐雙擴散對流之影響

鄭慶陽

南臺科技大學機械工程系

cycheng@stust.edu.tw

摘要

本文研究橫向磁場對雙分層多孔介質內垂直圓錐雙擴散對流之影響，此一垂直圓錐有定值之表面熱通量與質量通量。本文應用近似變數將統制偏微分方程式轉換為一組常微分方程式。再以三次樣線配置法求數值解。本文研究磁參數、熱分層參數、質量分層參數、路易士數與浮力比等參數對局部表面溫度與局部表面濃度之影響。對相似特例所得之解與與已發表之論文結果比較相當吻合。增加熱分層參數會減少局部表面溫度。局部表面濃度會隨著質量分層參數之增加而減少。而增大磁參數則會增大局部表面溫度與濃度。

關鍵詞：雙擴散、雙分層、自然對流、垂直圓錐、多孔介質、磁場

Magnetic Field Effects on Double Diffusive Convection from a Vertical Cone in Doubly Stratified Porous Media with Constant Surface Heat and Mass Fluxes

Ching-Yang Cheng

Department of Mechanical Engineering, Southern Taiwan University of Science and Technology

Abstract

This study examines the effects of transverse magnetic fields on the double diffusive convection about a vertical cone in a doubly stratified porous medium with constant surface heat and mass fluxes. This paper uses similarity variable to transform the governing partial differential equations into a system of ordinary differential equations, which are solved numerically by the cubic spline collocation method. The effects of magnetic parameters, thermal stratification parameters, mass stratification parameters, Lewis numbers, and buoyancy ratios on the local surface temperature and the local surface concentration are studied. A comparison with previously published results on the similar special cases shows an excellent agreement. The local surface temperature tends to decrease as the thermal stratification parameter is increased. Moreover, an increase in the mass stratification parameter leads to a decrease in the local surface concentration. Furthermore, increasing the magnetic parameter tends to increase the local surface temperature and concentration.

Keywords: Double Diffusion, Double Stratification, Natural Convection, Vertical Cone, Porous Medium, Magnetic Field

Received: Aug. 31, 2015; first revised: Feb. 26, 2016; accepted: March, 2016.

Corresponding author: C. -Y. Cheng, Department of Mechanical Engineering, Southern Taiwan University of Science and Technology, Tainan, Taiwan.

I. Introduction

The problems of the double diffusive natural convection in porous media saturated with fluids may be met in real world. There has been considerable interest in studying flows and heat and mass transfer rates of double diffusive natural convection in fluid-saturated porous media. The practical applications can be found in geothermal energy technology, petroleum recovery, and underground disposal of chemical and nuclear waste.

Bejan and Khair [1] examined the heat and mass transfer by natural convection in a porous medium. Lai [2] studied the heat and mass transfer by natural convection from a horizontal line source in saturated porous medium. Nakayama and Hossain [3] examined the heat and mass transfer by natural convection in a porous medium by integral methods. Singh and Queeny [4] studied the free convection heat and mass transfer along a vertical surface in a porous medium. Yih [5] examined the coupled heat and mass transfer by free convection over a truncated cone in porous media for variable wall temperature and concentration or variable heat and mass fluxes. Yih [6] examined the uniform transpiration effect on coupled heat and mass transfer in mixed convection about inclined surfaces in porous media for the entire regime. Cheng [7] presented the integral approach for heat and mass transfer by natural convection from truncated cones in porous media with variable wall temperature and concentration. Khanafer and Vafai [8] studied the double-diffusive mixed convection in a lid-driven enclosure filled with a fluid-saturated porous medium. Rathish Kumar et al. [9] studied the effect of thermal stratification on double-diffusive natural convection in a vertical porous enclosure. Cheng [10] studied the combined heat and mass transfer in natural convection flow from a vertical wavy surface in a power-law fluid saturated porous medium with thermal and mass stratification. Cheng [11] presented a nonsimilar boundary layer analysis of double-diffusive convection from a vertical truncated cone in a porous medium with variable viscosity. Mahdy [12] examined the effect of chemical reaction and heat generation or absorption on double-diffusive convection from a vertical truncated cone in porous media with variable viscosity.

There are many papers studied the effect of magnetic fields on the flow and heat transfer in porous medium saturated with electrically conducting fluid. Cheng [13] studied the effect of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media by an integral approach. Chamkha and Khaled [14] studied the hydromagnetic heat and mass transfer by mixed convection from a vertical plate embedded in a uniform porous medium. Cheng [15] studied the hydromagnetic natural convection heat and mass transfer from vertical surfaces with power law variation in wall temperature and concentration in porous media. Cheng [16] examined the hydromagnetic mixed convection heat and mass transfer from a vertical surface in a saturated porous medium Moreover, Muhaimin et al. [17] studied the thermophoresis and chemical reaction effects on non-Darcy MHD mixed convective heat and mass transfer past a porous wedge in the presence of variable stream condition.

The present work applies the similarity analysis to study the problem of heat and mass transfer by natural convection from a vertical cone with constant wall heat and mass fluxes embedded in doubly stratified porous media saturated with an electrically conducting fluid under the influence of a transverse magnetic field. The calculated results obtained herein are compared with the similar solutions calculated by previous studies to assess the accuracy of the present similarity method.

II. Analysis

Consider the boundary layer flow due to natural convection heat and mass transfer from a vertical cone of half angle A embedded in a porous medium saturated with an electrically conducting fluid subject to a uniform transverse magnetic field with thermal and mass stratification. The origin of the coordinate system is placed at the vertex of the full cone, with x being the coordinate along the surface of the cone measured from the origin and y being the coordinate perpendicular to the conical surface, as shown in Fig.1 The surface of the cone is held at a constant heat flux q_w while the porous medium temperature sufficiently far from the surface of the cone $T_\infty(x)$ varies with the streamwise coordinate. The mass flux of a certain constituent in the solution that saturated the porous medium is held at m_w near the surface of the cone while the concentration of this constituent in the solution that saturated the porous medium sufficiently far from the surface of the cone $C_\infty(x)$ varies with the streamwise coordinate.

The applied transverse magnetic field is assumed to be uniform and the magnetic Reynolds number is so small that induced magnetic field can be neglected. Further, the external electric field is assumed to be zero and the electric field due to polarization of charges is negligible. The fluid properties are assumed to be constant except for density variations in the buoyancy force term. The thermal and concentration boundary layers are assumed to be sufficiently thin compared with the local radius. Under the Boussinesq and the boundary layer approximations, the governing equations for the flow, heat and mass transfer within the boundary layer near the vertical cone can be written in two-dimensional Cartesian coordinates (x,y) as [5]

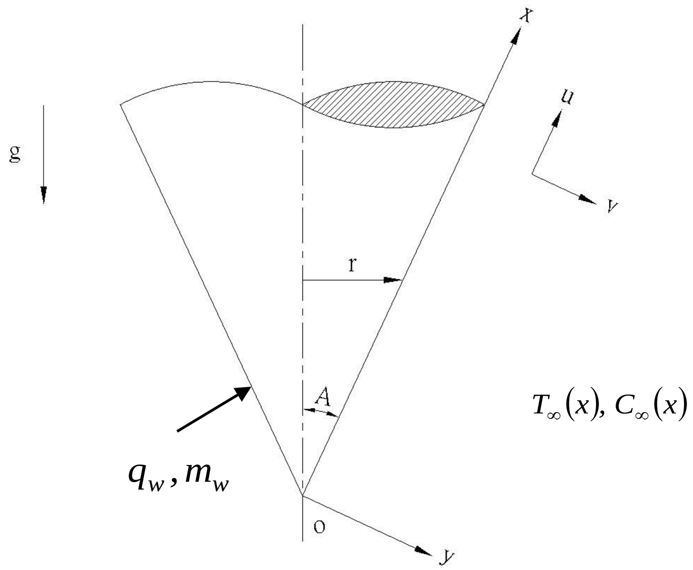

Fig.1 Flow configuration and coordinate system

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial y}=0 \tag{1}$$

$$\left(1+\frac{K\sigma B_0{}^2}{\varepsilon\mu}\right)u=\frac{\rho gK\cos A}{\mu}\left[\beta_T(T-T_\infty)+\beta_C(C-C_\infty)\right] \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

Here u and v are the volume-averaged velocity components in the x and y directions, respectively. T and C are the volume-averaged temperature and concentration, respectively. Properties μ, ρ, σ, and B_0 are the solution viscosity, density, electrical conductivity, and magnetic induction, respectively. K and ε are the permeability and the porosity of the porous medium, respectively. Furthermore, β_T and β_C are the coefficient of thermal expansion and the coefficient of concentration expansion, respectively. α and D are the thermal diffusivity and mass diffusivity of the porous medium, respectively. g is the gravitational acceleration.

The boundary conditions are given by

$$y=0: \quad v=0, \quad -k\left(\frac{\partial T}{\partial y}\right)_{y=0}=q_w, \quad -D\left(\frac{\partial C}{\partial y}\right)_{y=0}=m_w \tag{5}$$

$$y\to\infty: \quad u\to 0, \quad T\to T_{\infty,0}+\bar{S}_T x^{1/3}, \quad C\to C_{\infty,0}+\bar{S}_C x^{1/3} \tag{6}$$

where q_w, m_w, $T_{\infty,0}$ and $C_{\infty,0}$ are constants. $\bar{S}_T$ and $\bar{S}_C$ are the thermal stratification coefficient and mass stratification coefficient, respectively. Because the boundary layer thickness is small, the local radius to a point in the boundary layer r can be represented by the local radius of the vertical cone,

$$r=x\sin A \tag{7}$$

We introduce the similarity variables

$$\eta=\frac{y}{x}Ra^{1/3}, \quad f(\eta)=\frac{\psi}{\alpha\, rRa^{1/3}}, \quad \theta=\frac{k(T-T_\infty)Ra^{1/3}}{q_w x}, \quad \phi=\frac{D\,(C-C_\infty)Ra^{1/3}}{m_w x} \tag{8}$$

Where ψ is the stream function defined as:

$$u=\frac{1}{r}\frac{\partial\psi}{\partial y}, \quad v=-\frac{1}{r}\frac{\partial\psi}{\partial x} \tag{9}$$

and the Rayleigh number is given by

$$Ra=\frac{gK\beta_T x^2 q_w \cos A}{\nu\alpha k} \tag{10}$$

Upon using these variables, the boundary layer governing equations can be written in non-dimensional form as

$$\left(1+M^2\right)f'=\theta+N\phi \tag{11}$$

$$\theta''+\frac{5}{3}f\theta'-\frac{1}{3}f'\theta-\frac{1}{3}S_T f'=0 \tag{12}$$

$$\frac{\phi''}{Le}+\frac{5}{3}f\phi'-\frac{1}{3}f'\phi-\frac{1}{3}S_C f'=0 \tag{13}$$

where primes denote differentiation with respect to η.

Moreover, the Lewis number, buoyancy ratio, square of the magnetic parameter, thermal stratification parameter, and mass stratification parameter are respectively defined as

$$Le=\frac{\alpha}{D}, \quad N=\frac{\beta_C m_w k}{\beta_T q_w D}, \quad M^2=\left(K\sigma B_0^2\right)/(\varepsilon\mu), \quad S_T=\frac{k\bar{S}_T\left(gK\beta_T q_w\cos A\right)^{1/3}}{q_w(\nu\alpha k)^{1/3}},$$

$$S_C=\frac{D\bar{S}_C\left(gK\beta_T q_w\cos A\right)^{1/3}}{m_w(\nu\alpha k)^{1/3}} \tag{14}$$

The corresponding boundary conditions are given by

$$\eta=0: \quad f=0, \quad \theta'=-1, \quad \phi'=-1 \tag{15}$$

Table 1 Comparison of values of $\theta(0)$ and $\phi(0)$ for M=0, S_C =0, and S_T =0.

N	Le	$\theta(0)$		$\phi(0)$	
		Yih[5]	Present	Yih[5]	Present
4	1	0.6178	0.6178	0.6178	0.6178
4	10	0.9490	0.9489	0.2273	0.2273
4	100	1.0416	1.0416	0.0781	0.0781
1	1	0.8385	0.8384	0.8385	0.8384
1	10	1.0211	1.0211	0.2618	0.2618
1	100	1.0523	1.0522	0.0829	0.0829
0	1	1.0564	1.0564	1.0564	1.0564
0	10	1.0564	1.0564	0.2804	0.2804
0	100	1.0564	1.0564	0.0849	0.0849

$$\eta \to \infty: \quad f' \to 0, \ \theta \to 0, \ \phi \to 0 \tag{16}$$

In terms of the new variables, the Darcian velocities in the x- and y-directions can be expressed as

$$u = \frac{\alpha Ra^{2/3}}{x} f' \tag{17}$$

$$v = -\frac{\alpha Ra^{1/3}}{x}\left(\frac{5}{3} f - \frac{1}{3}\eta f'\right) \tag{18}$$

Results of practical interest are rates of heat and mass transfer from the wall to the fluid. The local Nusselt number of the cone can be written as

$$\frac{Nu}{Ra^{1/3}} = \frac{1}{\theta(0)} \tag{19}$$

In Eq. (19), $Nu = hx/k$ where h is the local convection heat transfer coefficient.

The local Sherwood number of the cone can be written as

$$\frac{Sh}{Ra^{1/3}} = \frac{1}{\phi(0)} \tag{20}$$

In Eq. (20), $Sh = h_m x/D$ where h_m is the local convection mass transfer coefficient.

III. Results and Discussion

In Table 1, The transformed governing equations, Eqs. (12) and (13), and the associated boundary conditions, Eqs. (15) and (16), can be solved by the cubic spline collocation method [18-20]. The velocity f' is calculated from the momentum equation, Eq. (11). Moreover, the Simpson's rule for variable grids is used to calculate the value of *f* at every position from the boundary conditions, Eqs. (15) and (16). The iteration process continues until the convergence criterion for all the variables, 10^{-6}, is achieved. Variable grids with 300 grid points are used in the η-direction. The optimum value of boundary layer thickness is used. To assess the accuracy of the solution, the present results are compared with those obtained by other researchers. Table 1 shows the numerical values of $\theta(0)$ and $\phi(0)$ for *M*=0, S_C =0, and S_T =0, the conditions for natural convection heat and mass transfer of a vertical cone of Newtonian fluids in porous media with constant wall heat and mass fluxes. It is shown that the present results agree well with the results reported by Yih [5].

Fig.2 shows the effects of the magnetic parameter M on the temperature profile $\theta(\eta)$ near a vertical cone. As the magnetic parameter is increased, the temperature distribution tends to increase, raising the thermal boundary layer thickness and thus decreasing the local Nusselt number. Fig.3 shows the effects of the magnetic parameter M on the concentration $\phi(\eta)$ near a vertical cone. As the magnetic parameter is increased, the concentration distribution tends to increase, raising the concentration boundary layer thickness and thus decreasing the local Sherwood number.

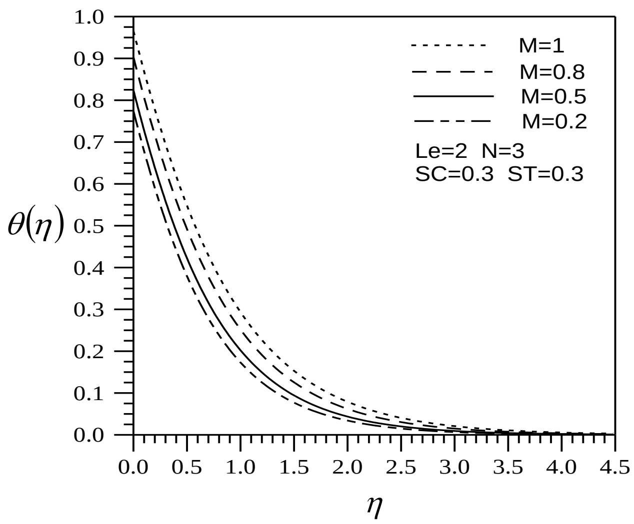

Fig.2 Effects of magnetic parameters on the temperature profile

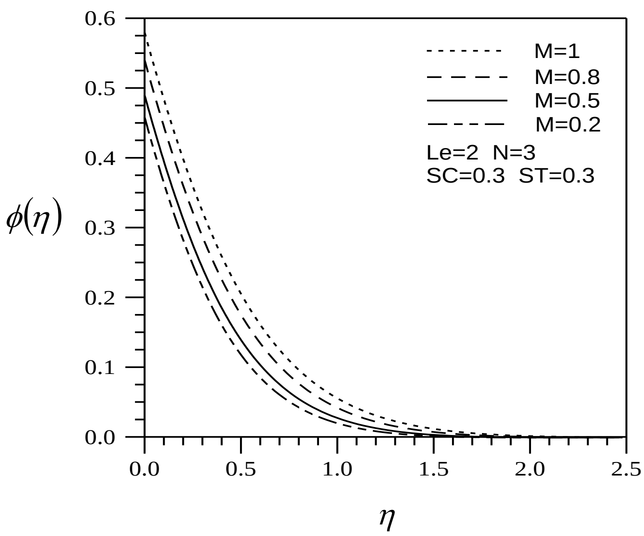

Fig.3 Effects of magnetic parameters on the concentration profile

Fig.4 shows the variation of the local surface temperature $\theta(0)$ with the magnetic parameter M for different values of Lewis numbers Le. It is shown in Fig.4 that increasing the magnetic parameter M tends to increase the local surface temperature, thus decreasing the local Nusselt number. That is because increasing the magnetic parameter retards the fluid flow, thus decreasing the local convective heat transfer coefficient between the porous medium and the conic surface. Comparing the three curves in Fig.4, we can deduce that as the Lewis number is increased, the local surface temperature tends to increase, thus decreasing the local Nusselt number.

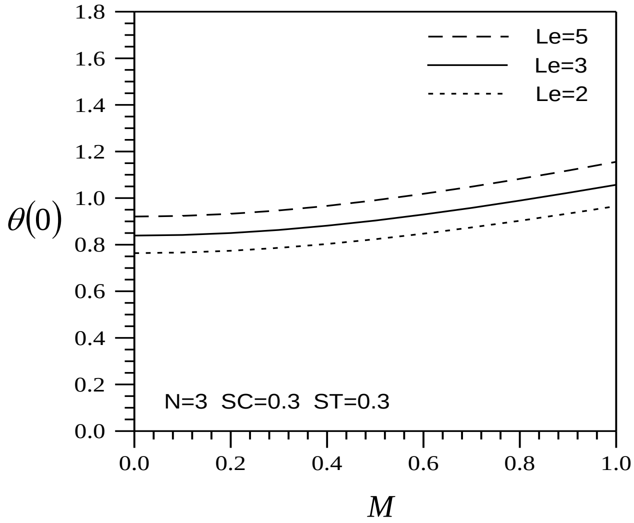

Fig. 4 Effects of magnetic parameters on the local surface temperature

Fig.5 depicts the local surface concentration $\phi(0)$ as a function of the magnetic parameter M for various values of Lewis numbers Le. Fig. 5 shows that an increase in the magnetic parameter tends to increase the local surface concentration, thus decreasing the local Sherwood number. That is because incresing the magnetic parameter decreases the fluid flow velocity and thus reduces the local convective mass transfer coefficient between the porous medium and the conic surface. Moreover, comparing the three curves in Fig.5, we can conclude that as the Lewis number is increased, the local surface concentration tends to decrease, thus increasing the local Sherwood number.

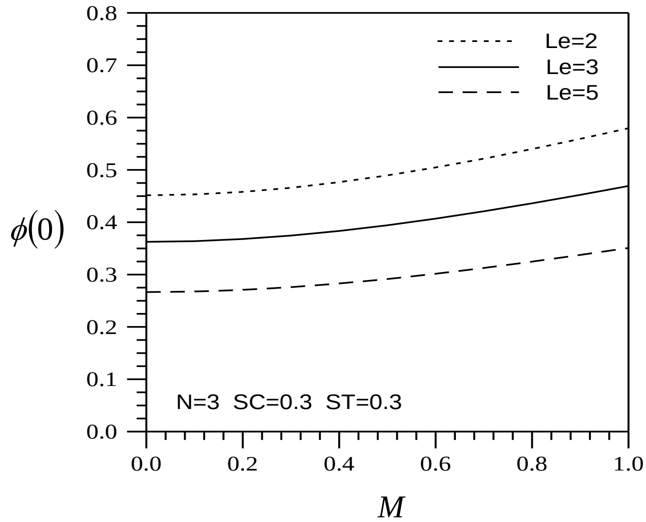

Fig. 5 Effects of magnetic parameters on the local surface concentration

Fig.6 plots the variation of the local surface temperature $\theta(0)$ with the thermal stratification parameter S_T for different values of buoyancy ratios N. It is shown in Fig.6 that raising the thermal stratification parameters leds to lower values of local surface temperatures, thus increasing the local Nusselt number. Moreover, comparing the four curves in Fig.6, we can deduce that an increase in the buoyancy ratio led to a decrease in the local surface temperature and thus an increase in the local Nusselt number in the natural convection flows, because increasing buoyancy ratio tends to accelerate the flow, thinning the thermal boundary layer and thus increasing the convective heat transfer coefficient between the porous medium and the conic surface.

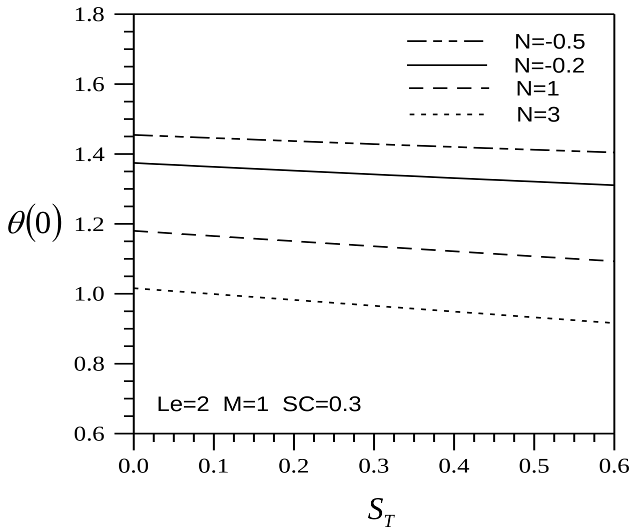

Fig. 6　Effects of thermal stratification parameters on the local surface temperature

Fig.7 plots the variation of the local surface temperature $\theta(0)$ with the mass stratification parameter S_T for different values of buoyancy ratios N. For positive values of buoyancy ratios, increasing the mass stratification parameter tends to increase the local surface temperature, thus decreasing the local Nusselt number, as shown in Fig.7 However, for negative values of buoyancy ratios, an increase in the mass stratification parameter tends to decrease the local surface temperature, thus increasing the local Nusselt number.

Fig.8 shows the local surface concentration $\phi(0)$ as functions of the mass stratification parameter S_C for different values of buoyancy ratios N. It is clearly shown in Fig.8 that increasing the mass stratification parameter tends to decrease the local surface concentration, thus increasing the local Sherwood number. Moreover, comparing the four curves in Fig.8, we can deduce that an increase in the buoyancy ratio led to a decrease in the local surface concentration and thus an increase in the local Sherwood number in the natural convection flows. That is because increasing buoyancy ratio tends to accelerate the flow, thinning the concentration boundary layer and thus increasing the convective mass transfer coefficient between the porous medium and the conic surface.

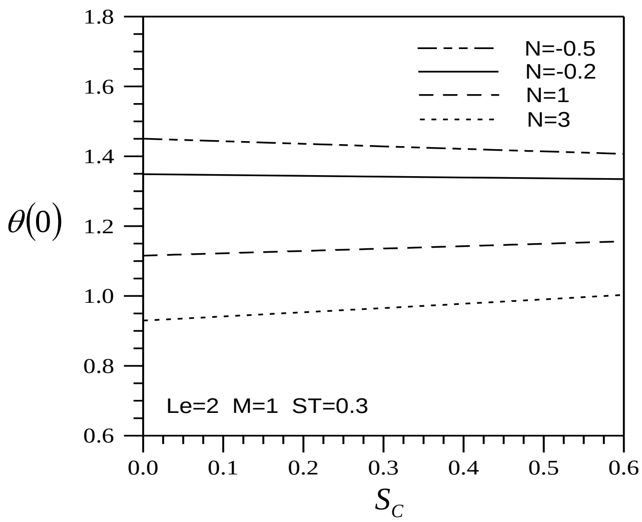

Fig. 7　Effects of mass stratification parameters on the local surface temperature

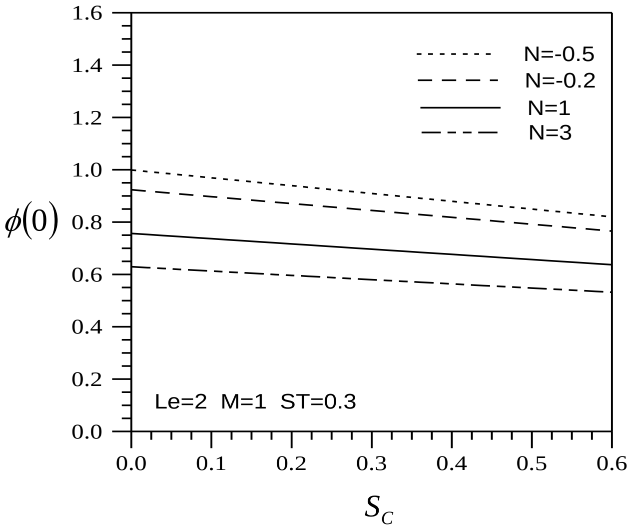

Fig. 8 Effects of mass stratification parameters on the local surface concentration

Fig.9 plots variation of the local surface concentration $\phi(0)$ with the thermal stratification parameters magnetic S_T for different values of buoyancy ratios N. It is clearly shown in Fig.9 that an increase in the thermal stratification parameter tends to increase the local surface concentration, thus decreasing the local Sherwood number.

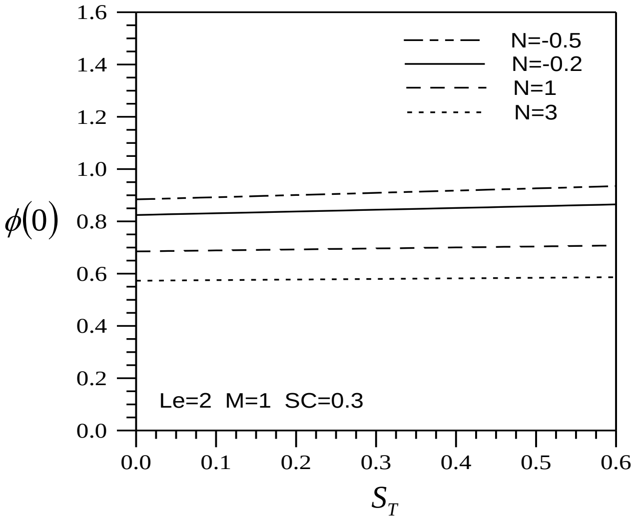

Fig. 9 Effects of thermal stratification parameters on the local surface concentration

IV. Conclusions

This work studies the magnetic field effects and the double stratification effects on the boundary layer flow due to natural convection heat and mass transfer over a vertical cone embedded in a porous medium with constant wall heat and mass fluxes. A similarity analysis is performed, and the obtained similar differential equations are solved by the cubic spline collocation method. The effects of the magnetic parameter, thermal stratification parameter, mass stratification parameter, Lewis number, and buoyancy ratio on the heat and mass transfer characteristics have been studied. As the thermal stratification parameter is increased, the local surface temperature decreases while the local surface concentration increases. An increase in the mass stratification parameter tends to decrease the local surface concentration. For positive values of buoyancy ratios, increasing the mass stratification parameter tends to increase the local surface temperature, thus decreasing the local Nusselt number. However, for negative values of buoyancy ratios, an increase in the mass stratification parameter tends to decrease the local surface temperature, thus increasing the local Nusselt number. Moreover, increasing the magnetic parameter decreases the local Nusselt number and the local Sherwood number. That is because incresing the magnetic parameter tends to decrease the fluid flow velocity and thus reduces the heat and mass transfer coefficients near the conic surface embedded in doubly stratified porous media.

References

[1] A. Bejan and K. R. Khair. (1985). Heat and mass transfer by natural convection in a porous medium, *International Journal of Heat and Mass Transfer, 28,* 909-918.

[2] F. C. Lai. (1990). Coupled heat and mass transfer by natural convection from a horizontal line source in saturated porous medium, *International Communications in Heat and Mass Transfer*, *17,* 489-499.

[3] A. Nakayama and M. A. Hossain. (1995). An integral treatment for combined heat and mass transfer by natural convection in a porous medium, *International Journal of Heat and Mass Transfer, 38,*761-765.

[4] P. Singh and Queeny. (1997). Free convection heat and mass transfer along a vertical surface in a porous medium, *Acta Mechanica*, *123,* 69-73.

[5] K. A. Yih. (1999). Coupled heat and mass transfer by free convection over a truncated cone in porous media: VWT/VWC or VHF/VMF, *Acta Mechanica, 137,* 83-97.

[6] K. A. Yih. (1999). Uniform transpiration effect on coupled heat and mass transfer in mixed convection about inclined surfaces in porous media: the entire regime, *Acta Mechanica, 132,* 229-240.

[7] C. Y. Cheng. (2000). Integral approach for heat and mass transfer by natural convection from truncated cones in porous media with variable wall temperature and concentration, *International Communications in Heat and Mass Transfer*, *27,*537-548.

[8] K. Khanafer and K. Vafai. (2002). Double-diffusive mixed convection in a lid-driven enclosure filled with a fluid-saturated porous medium, *Numerical Heat Transfer; Part A: Applications, 42*, 465-486.

[9] B. V. Rathish Kumar, P. Singh and V. J. Bansod. (2002). Effect of thermal stratification on double-diffusive natural convection in a vertical porous enclosure, *Numerical Heat Transfer, Part A: Applications, 41,* 421-447.

[10] C. Y. Cheng. (2009). Combined heat and mass transfer in natural convection flow from a vertical wavy surface in a power-law fluid saturated porous medium with thermal and mass stratification, *International Communications in Heat and Mass Transfer*, *36,* 351-356.

[11] C. Y. Cheng. (2009). Nonsimilar boundary layer analysis of double-diffusive convection from a vertical truncated cone in a porous medium with variable viscosity, *Applied Mathematics and Computation, 212,* 185-193.

[12] A. Mahdy. (2010). Effect of chemical reaction and heat generation or absorption on double-diffusive convection from a vertical truncated cone in porous media with variable viscosity, *International Communications in Heat and Mass Transfer*, *37,* 548-554.

[13] C. Y. Cheng. (1999). Effect of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media-an integral approach, *International Communications in Heat and Mass Transfer*, *26,*935-943.

[14] A. J. Chamkha and A. A. Khaled. (1999). Nonsimilar hydromagnetic simultaneous heat and mass transfer by mixed convection from a vertical plate embedded in a uniform porous medium, *Numerical Heat Transfer, Part A: Applications,36,* 327-344.

[15] C. Y. Cheng. (2005). An integral approach for hydromagnetic natural convection heat and mass transfer from vertical surfaces with power law variation in wall temperature and concentration in porous media, *International Communications in Heat and Mass Transfer*, *32*,204-213.

[16] C. Y. Cheng. (2007). Integral solution for hydromagnetic mixed convection heat and mass transfer from a vertical surface in a saturated porous medium, *WSEAS Transactions on Mathematics, 6*, 655-660.

[17] I. Muhaimin, I., R. Kandasamy and I. Hashim. (2009) Thermophoresis and chemical reaction effects on non-Darcy MHD mixed convection convective heat and mass transfer past a porous wedge in the presence of variable stream condition, *Chemical. Engineering Research and Design, 87,* 1527-1535.

[18] S. G. Rubin and R. A. Graves. (1975). Viscous flow solution with a cubic spline approximation, *Computers and Fluids, 3,* 1-36.

[19] P. Wang and R. Kahawita. (1983). Numerical integration of a partial differential equations using cubic spline, *International Journal of Computers and Mathematics*, *13*, 271-286.

[20] S. M. Ibrahim and N. B. Reddy. (2013). Similarity solution of heat and mass transfer for natural convection over a moving vertical plate with internal heat generation and a convective boundary condition in the presence of thermal radiation, viscous dissipation, and chemical reaction, *ISRN Thermodynamics*, *2013*, 790604.1-790604.10.